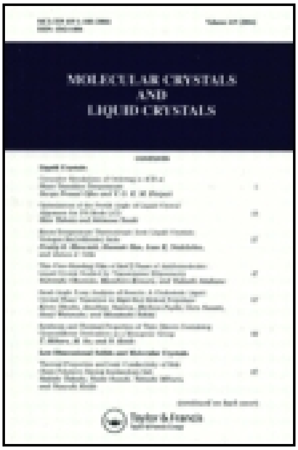

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Studies on the Optically Induced Twist Distortion In Nematics

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Studies on the Optically Induced Twist Distortion In Nematics†

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A general simple wave equation in nematic media aligned with an arbitrary twist distribution is obtained. An approximate solution for smoothly varying media is obtained and applied to the study of optically induced twist distortion. The adiabatic rotation of the polarization displayed by twisted nematics is a common argument to assume that such a distortion is not possible. In this paper, the impossibility of such a distortion is more accurately demonstrated.

Keywords: nematics, twist deformation, field-induced reorientation, wave equation, macroscopics, continuum elastic

1. INTRODUCTION

The main features, which have stimulated the interest in nematic liquid crystals, are their singular optical properties[1–3] and huge nonlinearities.[4,5] Some of their interest arises from the rotation of the polarization produced by nematic samples with twisted alignment. In this way, cholesteric and twisted nematics have been extensively studied as inhomogeneous anisotropic media. The studies usually deal with a linear variation of the twist angle, that is, helicoidal alignment with a constant pitch. To our knowledge, no general studies of arbitrary distributions of the twist angle have been performed yet.

Optically induced molecular reorientation has been widely studied over the past decade. In most instances homeotropic alignment has

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been employed,[6–9] whereas a few studies have considered hybrid aligned nematic cells.[10,11] Samples with planar alignment have not been considered since experimental evidence shows that reorientation is not achieved. This has been fundamented on Mauguin's adiabatic theorem,[6,11] according to which a smoothly varying twisted nematic rotates both ordinary and extraordinary polarizations about the director helix. As a consequence, if a molecular reorientation is produced, then the elastic energy increases, but the electromagnetic contribution to free energy remains constant. So, this situation should be unstable because the total energy has been increased. Mauguin's study, however, finds that a linear polarization incident in twisted nematic samples becomes elliptical with a small and variable ellipicity. There is then, a small variation of ordinary and extraordinary fields that could produce a weak reorientation if it modifies the free energy.

In this study we report a general and simple equations system that can be applied to any arbitrary twist-distorted nematic or similar inhomogeneous birefringent media. As an application of such a system a general solution for arbitrary smoothly distorted nematic media is obtained, and the possibility of an optically induced molecular reorientation is tested. This test, while not exact, is more accurate than the simple application of the adiabatic approximation. The result agrees with the assumption that reorientation is not produced in planar aligned nematics with normal incidence.

In the analysis of optically induced twist reorientation, the most peculiar feature is the presence of the optical axis in the same plane as the polarization. The electromagnetic problem has to be solved with the presence of both ordinary and extraordinary waves, even though the extraordinary index is constant. Every study reporting molecular reorientation consider, to our notice, structures where only extraordinary wave is present. However, the extraordinary index is variable. All the "exact" analytical solutions refer to normal incidence. Durbin *et al.*[6] seem to have considered oblique but straight ray paths, that is, refraction of light within the reoriented nematic film has not been taken into account. The oblique incidence problem has been solved by Ong[12] using the Geometrical Optics Aproximation;[13] however, only extraordinary wave is again considered. Geometrical Optics approximation had been previously used to obtain the field solution for normal incidence on nematic films.[7,14] In our study, only normal incidence of a plane wave is considered. This is a necessary assumption if we want to consider the twist distortion separately.

2. THE WAVE EQUATION WITHIN A TWIST DISTORTED NEMATIC

Let us consider the nematic as a superposition of birefringent layers whose optical axis show small angular displacements from each other within a plane parallel to the layers. A z axis perpendicular to the layers is chosen, and two different reference systems will be used (Figure 1). A fixed reference system will lay its x-y plane on the boundary of the first layer, and the x axis is taken as the origin in order to measure the twist angle of the optical axis, $\varphi(z)$. A second reference system is defined on each layer. It is constituted by the hereinafter named as parallel and perpendicular axes, being the first the optical axis, and the second its normal. Let us call Δz the thickness of each layer.

The electric field of the ordinary and extraordinary waves in a single layer (i.e., between z and $z + \Delta z$) can be expressed as

$$\begin{aligned} E_{\parallel}(z + \Delta z_-) &= E_{\parallel}(z) \exp ik_o n_{\parallel} \Delta z \\ E_{\perp}(z + \Delta z_-) &= E_{\perp}(z) \exp ik_o n_{\perp} \Delta z \end{aligned} \tag{1}$$

being $n_{\parallel}$ and $n_{\perp}$ the refractive indices of the layer, and $k_o = 2\pi/\lambda_o$ is the wave propagation constant in vacuum. These expressions can be rewritten with reference to the new axes in the next layer (see Figure 1.b)

$$\begin{aligned} E_{\parallel}(z+\Delta z_+) &= E_{\parallel}(z)\exp(ik_o n_{\parallel}\Delta z)\cos[\varphi(z+\Delta z) - \varphi(z)] - \\ &\quad - E_{\perp}(z)\exp(ik_o n_{\perp}\Delta z)\sin[\varphi(z+\Delta z) - \varphi(z)] \\ E_{\perp}(z+\Delta z_+) &= E_{\parallel}(z)\exp(ik_o n_{\parallel}\Delta z)\sin[\varphi(z+\Delta z) - \varphi(z)] + \\ &\quad + E_{\perp}(z)\exp(ik_o n_{\perp}\Delta z)\cos[\varphi(z+\Delta z) - \varphi(z)] \end{aligned} \tag{2}$$

In the limit when $\Delta z \rightarrow 0$

$$\begin{aligned} E_{\parallel}(z) + \frac{dE_{\parallel}}{dz}\Delta z &= E_{\parallel}(z)[1 + ik_o n_{\parallel}\Delta z] - E_{\perp}(z)\frac{d\varphi}{dz}\Delta z \\ E_{\perp}(z) + \frac{dE_{\perp}}{dz}\Delta z &= E_{\parallel}(z)\frac{d\varphi}{dz}\Delta z + E_{\perp}(z)[1 + ik_o n_{\perp}\Delta z] \end{aligned} \tag{3}$$

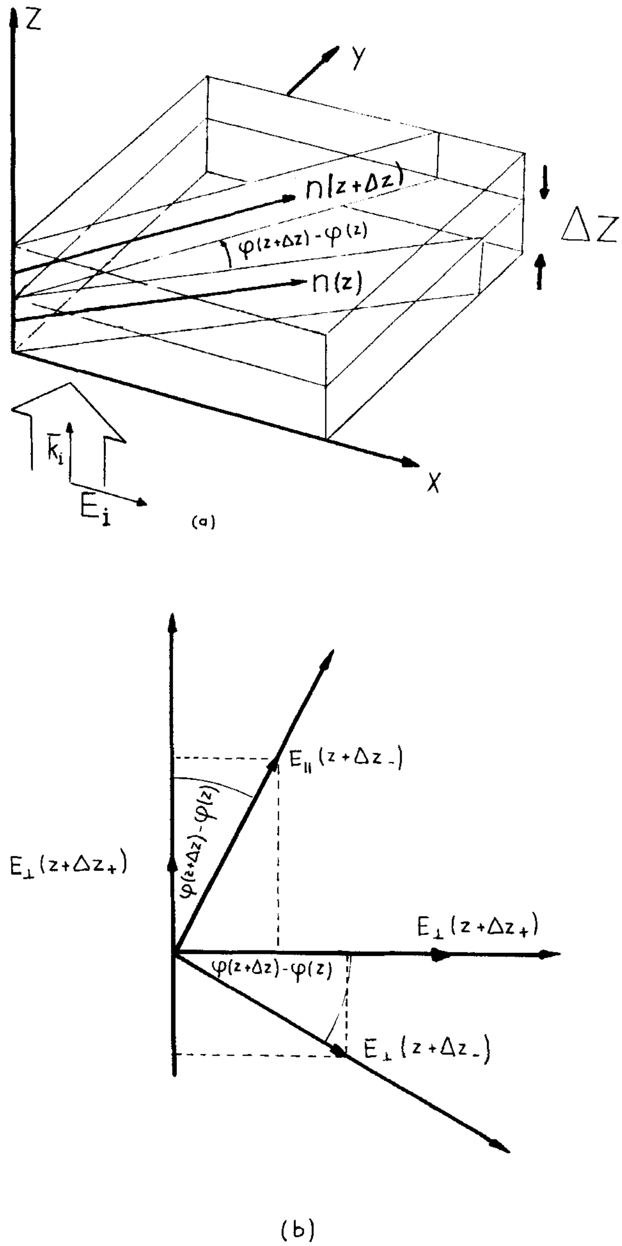

FIGURE 1 Description of the framework. (a) Elemental layer. (b) Rotation of the axes at the boundary between two layers.

In the variable reference system, therefore, continuously varying media show the following wave equation:

$$dE/dz = [M]E \tag{4}$$

where

$$E = [E_\parallel(z), E_\perp(z)] \tag{5}$$

$$[M] = \begin{bmatrix} i\,k_o\,n_\parallel & -d\varphi/dz \\ d\varphi/dz & i\,k_o\,n_\perp \end{bmatrix} \tag{6}$$

This is a general first order differential equations system which stands for all kinds of twist distorted structures. So, it can be particularized to describe a cholesteric or a twisted nematic cell; in these cases all the elements in (6) are constants (see Appendix). The equation obtained is as simple as those used in studies "ad hoc"[2,15]; moreover, it yields a more complete solution than the result of methods based in the Jones calculus[3] or Poincarè's sphere,[16] which only give information about the polarization state. However, only one direction has been considered for the wave propagation. Hence, the study is not valid in periodic structures whose pitch is comparable to the wavenlength, since Bragg reflections should appear.

From equation (4) we can obtain

$$E_\perp = -\frac{1}{\left(\frac{d\varphi}{dz}\right)}\frac{dE_\parallel}{dz} + \frac{i\,k_o\,n_\parallel}{\left(\frac{d\varphi}{dz}\right)}E_\parallel \tag{7}$$

Substituting in the other equation yields

$$\frac{d^2E_\parallel}{dz^2} - \left[i\,k_o(n_\parallel + n_\perp) + \frac{\frac{d^2\varphi}{dz^2}}{\frac{d\varphi}{dz}}\right]\frac{dE_\parallel}{dz} + \left[\left(\frac{d\varphi}{dz}\right)^2 + ik_on_\parallel\frac{\frac{d^2\varphi}{dz^2}}{\frac{d\varphi}{dz}} - k_o^2n_\parallel n_\perp\right]E_\parallel = 0 \tag{8}$$

This is the general wave equation for the extraordinary wave in a arbitrary continuously twist distorted NLC. The analytical solution of this equation is quite complicated. So let us make an approximation which leads to a simple solution.

3. SMOOTHLY VARYING DIRECTOR APPROACH

The Poincarè's sphere analysis of twisted nematic cells shows[16] that the polarization state through the cell is slightly elliptical, its major axis being along the nematic director. The ellipticity varies periodically, and the polarization becomes completely linear at equally spaced points. If molecular reorientation is produced by an optical field, a similar variation is to be expected. There would be reorientation in those regions where a normal component is present; and there would not, in those points where the optical field is parallel to the director. The reorientation, however, should be very weak. Therefore, if we assume that there is no initial distortion (i.e. a planar aligned cell), it can be expected that

$$(d\varphi/dz)^2 << k_o^2 \tag{9}$$

Then, equation (8) can be approximated to

$$d^2E/dz^2 - [ik_o(n_\parallel + n_\perp) + \varphi''(z)/\varphi'(z)]\,dE/dz + [ik_o n_\parallel \varphi''(z)/\varphi'(z) - k_o^2 n_\parallel n_\perp]E = 0 \tag{10}$$

That is, the term $[\varphi'[(z)]^2$ (the smallest one in the described conditions) is neglected. This approach can be tested to be met from the solution. Equation (10) can be solved with the simple change of variables:

$$E_\parallel(z) = \psi(z)\exp(ik_o n_\parallel z)$$

resulting

$$\psi'' + \left[ik_o n_a - \frac{\varphi''}{\varphi'}\right]\psi' = 0 \tag{11}$$

This is a first order linear equation in ψ', which yields an electric field

$$E_{\parallel}(z) = e^{ik_o n_{\parallel} z}\left[E_2 + E_1 \int_o^z \varphi'(z) e^{ik_o n_a z}\, dz\right] \tag{12}$$

where $n_a = n_{\parallel} - n_{\perp}$. The integration constants can be determined from the boundary conditions (see Figure 1):

$$E_{\parallel}(z) = E_o \cos \varphi(o)$$

$$E_{\perp}(z) = E_o \sin \varphi(o)$$

It has to be noticed that substitution of (12) in (7) to obtain the ordinary field, gives $E_{\perp} = E_1 \exp ik_o n_{\perp} z$. So ordinary wave is obtained less accurately than the extraordinary one. However, the expression is valid if we are only interested for the value at the boundary. Let us call $\varphi(O) = \varphi_o$. Substitution of the integration constants gives

$$E_{\parallel}(z) = E_o \exp(ik_o n_{\parallel} z)\left[\cos\varphi_o + \sin\varphi_o \int_o^z \varphi'(z) \exp(ik_o n_a z) dz\right] \tag{13}$$

This expression gives an estimation of the optical field of the extraordinary wave at an arbitrarily twist-distorted nematic; its accuracy is better than the mere extrapolation of Mauguin's theorem. In any case, if the exact solution is required, equation (8) has to be solved. This solution can be simple for some particular angle functions $\varphi(z)$.

4. FREE ENERGY OF THE SYSTEM

In the study of optically induced molecular reorientations a question arises about the way the optical energy contributes to the energy of the system. A comparative discussion of this problem from several works is made by Ong.[6] The summary of this discussion is: (1) Free energy must be derived in a self-consistent way from Maxwell's equations, and (2), in spite of the absence of a magnetic anisotropy, there is a variable contribution of magnetic energy, due to variations of the field produced by reorientation of the director. However he preferred, as previously did Durbin *et al.*,[5] to compute the optical energy density from the optical intensity. In this study we have derived the optical energy density from the field amplitude within the nematic. Two advantages are obtained from this outline. First, constant terms are not introduced in the optimization of the free energy. Second, the optical energy density term is directly calculated from the optical field obtained in the previous section, and it is not necessary to develop further calculations in order to obtain the optical intensity in terms of the field amplitude.

In the above described fixed coordinate framework, the director should be

$$\hat{n} = (\cos\varphi, \sin\varphi, 0) \tag{14}$$

and so the dielectric tensor

$$[\epsilon] = \begin{bmatrix} \epsilon_\perp + \epsilon_a \cos^2\varphi & \epsilon_a \cos\varphi \sin\varphi & O \\ \epsilon_a \cos\varphi \sin\varphi & \epsilon_\perp + \epsilon_a \sin^2\varphi & O \\ O & O & \epsilon_\perp \end{bmatrix} \tag{15}$$

In this way, for a normally incident plane wave **D** and **E** are both parallel to the X-Y plane. If we assume that $[\epsilon]$ varies smoothly with z, then magnetic and electric energy densities coincide, and the total optical intensity density is

$$W_{em} = 2\,W_e = \langle\, \mathbf{E} \cdot \mathbf{D}^* \rangle = \epsilon_\perp\, (\langle E_x E_x^* \rangle + E_y E_y^* \rangle)$$

$$+\ \epsilon_a\, (E_x\cos\varphi + E_y\sin\varphi)(E_x\cos\varphi + E_y\sin\varphi)^* \tag{16}$$

The first term is constant with φ, so it does not need to be included to minimize the energy. The variable part of the optical energy is

$$F_{\text{opt}} = \epsilon_a\, \langle (\mathbf{E}\cdot\hat{\mathbf{n}})(\mathbf{E}\cdot\hat{\mathbf{n}})^* \rangle$$

For a harmonic wave, $E_\parallel$ being the extraordinary wave phasor, we have

$$F_{\text{opt}} = \epsilon_a E_\parallel E_\parallel^* / 2 \tag{17}$$

The variable term of the elastic energy density is given by the elastic continuum theory as

$$F_{\text{elas}} = k_{22}[\hat{\mathbf{n}} \cdot \nabla \times \hat{\mathbf{n}}]^2/2$$

assuming that only twist distortion is present. In our problem the director varies only with z, given by (14) so that

$$F_{\text{elas}} = k_{22}\,(d\varphi/dz)^2/2 \tag{18}$$

So the free energy per unit area, in a nematic film with thickness w, is

$$\mathcal{F} = 1/2 \int_o^w [k_{22}(d\varphi/dz)^2 - \epsilon_a\,|E_{\|}|^2]dz \tag{19}$$

Substituting E from expression (13), it results

$$\mathcal{F} = \frac{1}{2}\int_o^w \Bigg[k_{22}\left(\frac{d\varphi}{dz}\right)^2 - E_o\epsilon_a \sin 2\varphi_o \int_o^z \frac{d\varphi}{dz}\cos k_o n_a z dz - E_o^2\epsilon_a \sin^2\varphi_o \times \left\{ \left[\int_o^z \frac{d\varphi}{dz}\cos k_o n_a z\, dz\right]^2 + \left[\int_o^z \frac{d\varphi}{dz}\sin k_o n_a z\, dz\right]^2 \right\} \Bigg] dz \tag{20}$$

In (20) we have z to express the variable in two different integration levels. This mathematical misuse has been preferred to introduce new variables which could be more confusing.

5. SOLUTION OF THE EULER EQUATION

In order to optimize the functional (20) the Euler equation to be solved, is

$$O = k_{22}\frac{d^2\varphi}{dz^2} - E_o^2\epsilon_a \sin\varphi_o \cos\varphi_o \cos k_o n_a z - E_o^2\epsilon_a \sin^2\varphi_o \cdot \frac{d}{dz}\left[\frac{\sin k_o n_a z}{k_o n_a}\int_o^z \frac{d\varphi}{dz}\cos k_o n_a z dz - \frac{\cos k_o n_a z}{k_o n_a}\int_o^z \frac{d\varphi}{dz}\sin k_o n_a z dz\right] \tag{21}$$

A first integration of this equation gives

$$\frac{d\varphi}{dz} = C_1 + \frac{1}{\xi^2}\left[\frac{\sin\varphi_o\cos\varphi_o}{k_o n_a}\sin k_o n_a z + \frac{\sin^2\varphi_o}{k_o n_a} \left\{\sin k_o n_a z \int_o^z \frac{d\varphi}{dz}\cos k_o n_a z\, dz - \cos k_o n_a z \int_o^z \frac{d\varphi}{dz}\sin k_o n_a z\, dz\right\}\right] \tag{22}$$

Where ξ is the optical field coherence length

$$\xi = (k_{22}/\epsilon_a)^{1/2}/E_o \tag{23}$$

and the integration constant C_1 can be proved to be

$$C_1 = d\varphi/dz|_{z=0} \tag{24}$$

By means of two successive derivates, the integrals can be eliminated, and the following differential equation is obtained:

$$d^3\varphi/dz^3 + (k_o^2n_a^2 - \sin^2\varphi_o/\xi^2)\, d\varphi/dz = k_o^2n_a^2\, C_1 \tag{25}$$

This differential equation is second order in $\varphi'(z)$, and linear with constant coefficients. Let us call

$$\alpha = (k_o^2n_a^2 - sin^2\, \varphi_o/\xi^2)^{1/2} \tag{26}$$

The general solution is

$$\varphi'(z) = C_2 \cos(\alpha z + \Phi) + k_o^2n_a^2/\alpha^2\, C_1 \tag{27}$$

From equation (21)

$$\varphi''(0) = \sin\varphi_o \cos\varphi_o/\xi^2 \tag{28}$$

With conditions (28) and (24) C_2 and C_1 can be determined:

$$\varphi'(z) = \frac{k_o^2n_a^2}{\alpha\, \tan\varphi_o\, \tan\phi} - \frac{\sin\varphi_o \cos\varphi_o}{\alpha\xi^2}\frac{\cos[\alpha z+\phi]}{\sin\phi} \tag{29}$$

The integration of (29) and the boundary condition $\varphi(O) = \varphi_o$ give

$$\varphi(z) = \varphi_o + \frac{k_o^2\, n_a^2}{\alpha\tan\varphi_o\tan\phi z} + \frac{\sin\varphi_o \cos\varphi_o}{\alpha^2\xi^2}\left(1 - \frac{\sin(\alpha z+\phi)}{\sin\phi}\right) \tag{30}$$

The integration constant ϕ must be determined from the boundary condition at the opposite surface $z=w$. In the general case, for a planar cell with an initial twist $\Delta\varphi$, we have $\varphi(w)=\varphi_o+\Delta\varphi$, giving

$$\cot\phi = \frac{\Delta\varphi\,\alpha^2\tan\varphi_o - (1-\cos\alpha w)\dfrac{\sin^2\varphi_o}{\xi^2}}{k_o^2 n_a^2 \alpha w - \dfrac{\sin^2\varphi_o}{\xi^2}\sin\alpha w} \tag{31}$$

This expression is only valid for small values of the initial twist $\Delta\varphi$. Otherwise $d\varphi/dz$ has a constant term, and the approximation (9) cannot be made.

In the case of a planar sample ($\Delta\varphi=0$),

$$\tan\phi = -\frac{\dfrac{k_o^2 n_\sigma^2}{\sin^2\varphi o}\zeta^2 - \sin\alpha w}{1-\cos\alpha w} \tag{32}$$

It can be easily seen that ϕ is always negative and usually it is near $-\pi/2$. On the contrary, choosing the negative value of the square root in (26), ϕ would have been positive, resulting on the same expression for $\varphi(z)$.

6. CALCULATIONS AND DISCUSSION

Substitution of expression (30) in (13) gives

$$E_{\|}(z) = E\cos\varphi_o\left[\frac{\sin(\alpha z+\phi)}{\sin\phi} + i\frac{k_o n_a}{\alpha\tan\phi}\left(1-\frac{\cos(\alpha z+\phi)}{\cos\phi}\right)\right]e^{ik_o n_\perp z} \tag{33}$$

Notice that the phase is not $k_o n_\perp z$, but

$$k_o n_\perp z + \tan^{-1}\left[\frac{k_o n_a}{\alpha}\frac{\cos\phi - \cos(\alpha z+\phi)}{\sin(\alpha z+\phi)}\right]$$

In particular it can be proved that, in case of no reorientation ($\xi\to\infty$)

$$E_{\|}(z) = E_o\cos\varphi_o \exp ik_o n_{\|} z$$

We have performed some calculations of functions (30) and (33). The relationship between the optical intensity and the coherence length has to take into account the presence of both ordinary and extraordinary waves. We can write

$$I_{\parallel} = E_{\parallel}^2 \, n_{\parallel}/2\eta_o$$

$$I_{\perp} = E_{\perp}^2 \, n_{\perp}/2\eta_o$$

where $\eta_o = 120\,\pi$ is the vacuum electromagnetic impedance.

For $\varphi_o = \pi/4$, $E_{\parallel}(O) = E_{\perp}(O)$, and we can find

$$\xi^2 = \frac{k_{22}\,(n_{\parallel}+n_{\perp})}{4\eta_o\,\varepsilon_a\,I} = \frac{k_{22}\cdot 9\cdot 10^9}{120\,I}\,\frac{n_{\parallel}+n_{\perp}}{n_{\parallel}^2 - n_{\perp}^2} \tag{34}$$

Figure 2 shows the results of a 20 μm film of MBBA at λ = 514.5nm. The calculation parameters are shown in Table I. Figure 3 shows the variation with the initial angle φ_o between the input polarization and the director at the surfaces. The calculation parameters are presented in Table II. Finally Figure 4 and Table III show a comparison between MBBA and PCB.

These results are not, however, a physical solution of the problem. The calculation of free energy according to (19) gives an energy value higher than that obtained when reorientation vanishes. In case of no reorientation

$$\mathcal{F}_{nr} = -\frac{1}{2}\,\varepsilon_a\,E_o^2\cos^2\varphi_o\,w \tag{35}$$

TABLE I

Several optical intensities used in the calculations of Figure 2. Field coherence length, amplitude of the distortion and energy are indicated. ($\varphi_o = \pi/4$, $\lambda = 514.5$nm)

Curve	$I(kw/cm^2)$	ξ (μm)	φ_m see expression (39)	ΔF_N see exp. (37)
1	20	2.26	1.109×10^{-2}	-1.183×10^{-5}
2	10	3.2	5.502×10^{-3}	-6.095×10^{-6}
3	5	4.53	2.738×10^{-3}	-3.059×10^{-6}

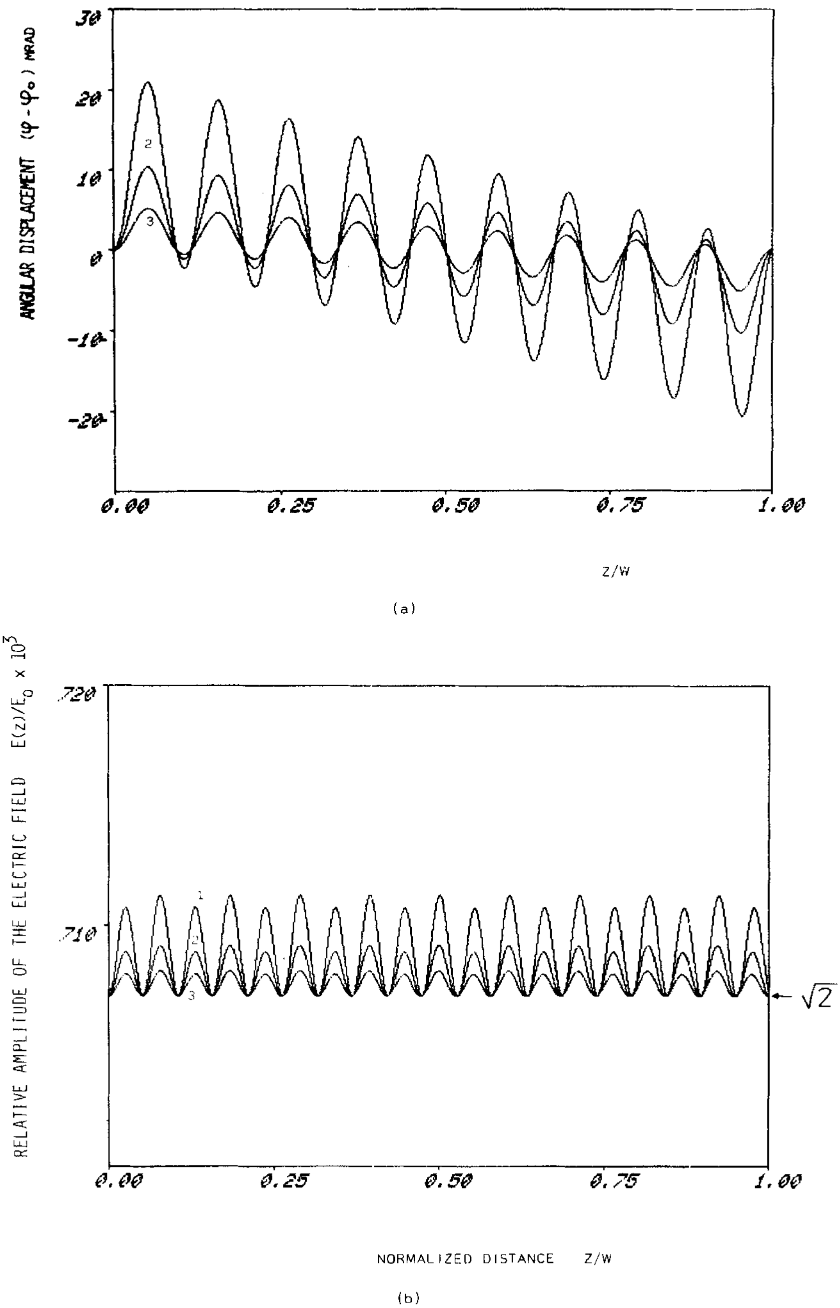

FIGURE 2 Calculations at several optical intensities (see Table I): (a) angular displacement $\varphi(z)-\varphi(o)$; b amplitude of the electric field.

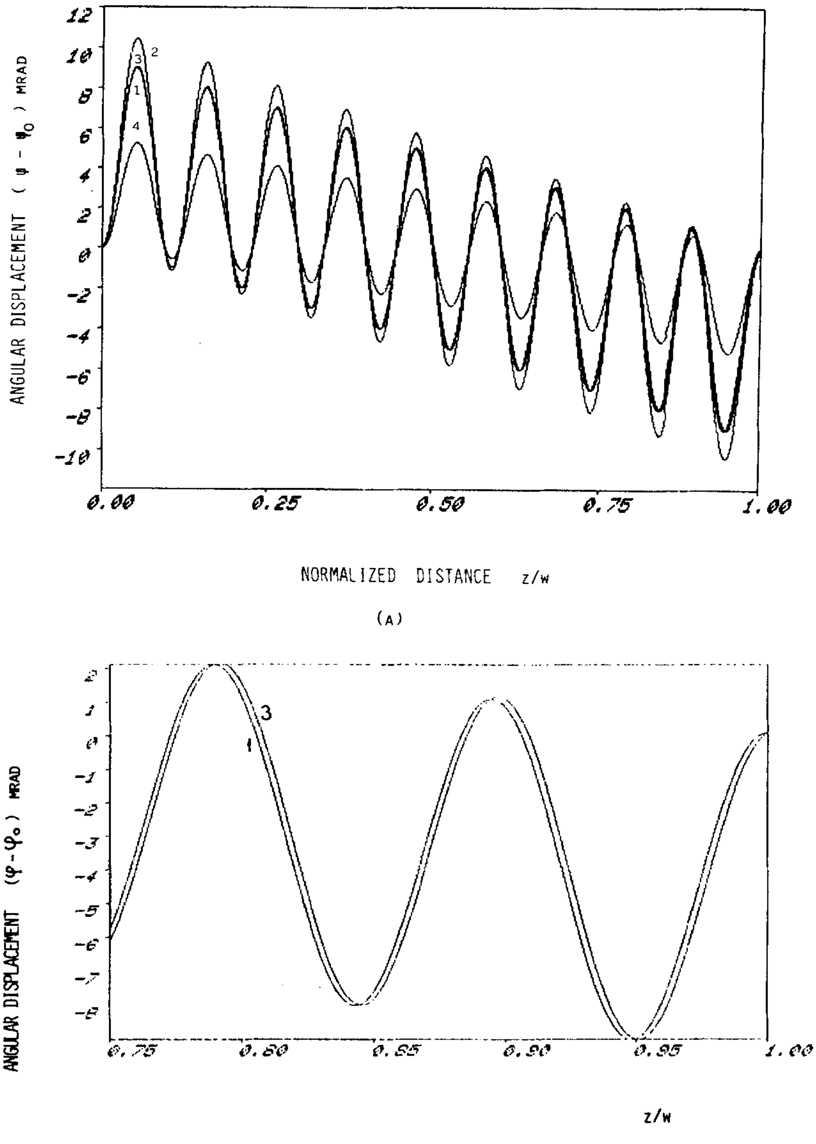

FIGURE 3 Calculations at several angles between the input polarization and the molecular orientation at the boundaries of the cell. Angular displacement versus distance (see Table II).

TABLE II

Several angles between incident wave polarization and director orientation at the boundary used in the calculations of Figure 3. Amplitude of the distortion and normalized free energy are also indicated. ($\xi = 3.2\mu m$, $\lambda = 51145nm$)

Curve	φo	φm (rad)	ΔF_n
1	30°	4.752×10^{-3}	-3.065×10^{-6}
2	45°	5.502×10^{-3}	-6.095×10^{-6}
3	60°	4.778×10^{-3}	-9.028×10^{-6}
4	75°	2.764×10^{-3}	-1.108×10^{-5}

$$\mathcal{F}_{wr} = \frac{1}{2}\,\varepsilon_a E_o^2\,\frac{\cos^2\varphi o}{\sin^2\phi}\left[\frac{ko^2na^2\,\xi^2\cos\phi}{\sin^2\varphi o} - 1\right] w \tag{36}$$

A normalized energy increase can be defined as

$$\Delta F_n = \frac{\mathcal{F}_{nr} - \mathcal{F}_{wr}}{|\mathcal{F}_{nr}|} \tag{37}$$

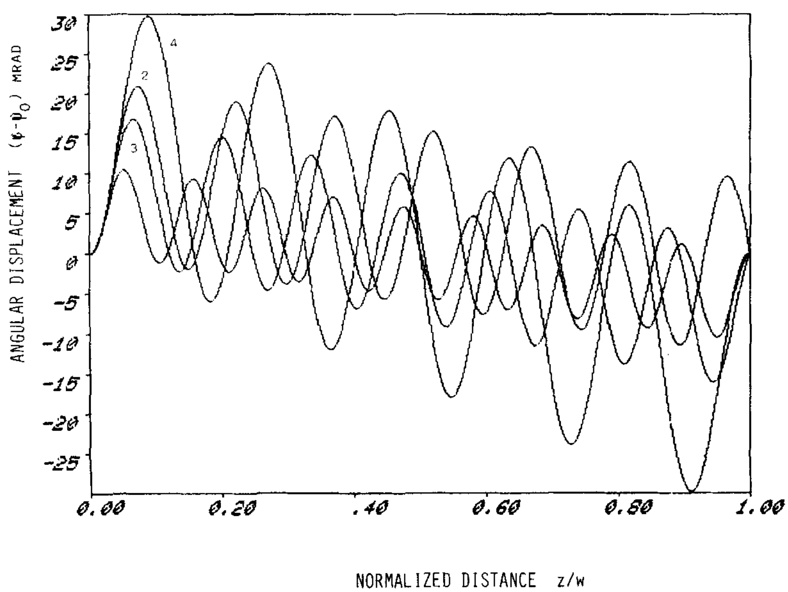

FIGURE 4 Comparison between two liquid crystals (PCB and MBBA). See Table III.

TABLE III

Refractive indices at two wavelengths for nematic liquid crystals MBBA and PCB used in the calculations of Figure 4. Amplitude of distortion and normalized free energy are also indicated. In all situations the optical field coherence length has been 3.2 microns. The refractive indices have been taken from Ref.[1]

Curve	Liquid Crystal	(Å)	n_e	n_o	φm(rad)	F_n
1	MBBA	5145	1.8062	1.5616	5.502×10^{-3}	-6095×10^{-6}
2	MBBA	6328	1.7582	1.5443	1.094×10^{-2}	-8.128×10^{-6}
3	PCB	5145	1.7360	1.5442	8.980×10^{-3}	-1.439×10^{-5}
4	PCB	6328	1.7063	1.5309	1.636×10^{-2}	-5.307×10^{-5}

this gives

$$\Delta F_N = -\alpha^2\xi^2 \cot an^2 \Phi/\sin^2\varphi_o \qquad (38)$$

which is always a negative value.

Extracting the amplitude of the reorientation

$$\varphi_m = \sin\varphi_o \cos\varphi_o/\alpha^2\xi^2 \qquad (39)$$

as a common factor in the expression of $\varphi'(z)$, and letting φ_m to vary, the obtained free energy describes a parabolic curve whose maximum is the value given by (39).

As a conclusion, this situation is unstable, thus confirming that there is not optically induced twist distortion.

7. CONCLUSIONS

We have derived a general wave equation of the optical field in a twist distorted nematic with an arbitrary angle distribution $\varphi(z)$. This equation has been used to test the possibility of optically induced twist distortion. In previous studies by other authors, the adiabatic condition has been assumed, and the absence of such a reorientation has been justified in that way. Nevertheless, a question arises on a weak reorientation being still present, and neglected by the adiabatic approximation. In our study more terms have been allowed in our general equation. The solution, however, confirms that such a distortion cannot be produced. The adiabatic study assumes that there is not an amplitude variation and keeps the most significant terms of the wave equation. A more complete study should include the remaining term which would add a neglective variation without an appreciable modification of the results. In planar homogeneous samples, we conclude, there is not optical reorientation with normal incidence. In case of initial twist considerable distortion, (i.e., no adiabatic rotation is produced) the exact study must be developed, but the analytical solution of the wave equation can not be easily obtained.

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APPENDIX : OPTICAL FIELD FOR LINEARLY VARYING TWIST ANGLE

In the case of a chiral nematic or a cholesteric,

$$\frac{d\varphi}{dz} = \frac{2\pi}{p} \tag{A.1}$$

being p the pitch of the helix. Then, equations (4) result

$$\begin{aligned} \frac{dE_{\parallel}}{dz} &= ik_o n_{\parallel} E_{\parallel} - \frac{2\pi}{p} E_{\perp} \\ \frac{dE_{\perp}}{dz} &= ik_o n_{\perp} E_{\perp} + \frac{2\pi}{p} E_{\parallel} \end{aligned} \tag{A.2}$$

Under Mauguin's condition $p >> \lambda$, both equations are independent and both ordinary and extraordinary wave amplitudes are independent of z, each one having its corresponding propagation constant

$$\begin{aligned} E_{\parallel} &= E_{op} \exp(ik_o n_{\parallel} z) \\ E_{\perp} &= E_{on} \exp(ik_o n_{\perp} z) \end{aligned} \tag{A.3}$$

i.e., the polarization is rotated. Under the opposite condition $p<<\lambda$

$$\begin{aligned} \frac{d^2E_\parallel}{dz^2} + \left(\frac{2\pi}{p}\right)^2 E_\parallel &= 0 \\ \frac{d^2E_\perp}{dz^2} + \left(\frac{2\pi}{p}\right)^2 E_\perp &= 0 \end{aligned} \tag{A.4}$$

Now the propagation vector is the same for both waves and independent of the wavelength

$$\begin{aligned} E_\parallel &= E_{op} \exp i \frac{2\pi}{p} z \\ E_\perp &= E_{on} \exp i \frac{2\pi}{p} z \end{aligned} \tag{A.5}$$

In the general case (p and λ comparable), the wave equation, by substituting in (8) is

$$E''_\parallel(z) - ik_o\,(n_\parallel + n_\perp)E'_\parallel(z) + \{(2\pi/p)^2 - k_o^2 n_\parallel n_\perp\}E_\parallel(z) = 0 \tag{A.6}$$

and the solution is

$$E_\parallel = E_{opi} \exp ik_o \left[\bar{n} + \sqrt{\left(\frac{n_a}{2}\right)^2 + \left(\frac{\lambda}{P}\right)^2}\,\right] z + E_{op2} \exp ik_o \left[\bar{n} - \sqrt{\left(\frac{n_a}{2}\right)^2 + \left(\frac{\lambda}{P}\right)^2}\,\right]_a z \tag{A.7}$$

where $\bar{n} = (n_\parallel + n_\perp)/2$ and $n_a = n_\parallel - n_\perp$. As indicated above, this solution is not valid when $p \simeq \lambda$.

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